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## Distributed traffic control for reduced fuel consumption and travel time in transportation networks

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### Recommended Citation

Dai, Ran; Dong, Jing; and Sharma, Anuj, "Distributed traffic control for reduced fuel consumption and travel time in transportation networks" (2015). *Civil, Construction and Environmental Engineering Conference Presentations and Proceedings*. 44.

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### Abstract

This paper proposes a distributed framework for optimal control of vehicles in transportation networks. The objective is to reduce the balanced fuel consumption and travel time through hybrid control on speed limit and ramp metering rate. The dual decomposition theory associated with the subgradient method is then applied in order to decompose the optimal control problem into a series of suboptimal problems and then solve them individually via networked road infrastructures (RIs). Coordination among connected RIs is followed in each iteration to update the individual controls. An example is demonstrated to verify the reduction in terms of fuel consumption and travel time using the proposed approach.

### Keywords

Fuels, Vehicles, Mathematical model, Vehicle dynamics, Optimization, Optimal control

### Disciplines

Civil and Environmental Engineering | Transportation Engineering

### Comments

This is a manuscript of a proceeding published as Dai, Ran, Jing Dong, and Anuj Sharma. "Distributed traffic control for reduced fuel consumption and travel time in transportation networks." In European Control Conference (ECC), 2015, pp. 2658-2663. IEEE, 2015. DOI: [10.1109/ECC.2015.7330939](https://doi.org/10.1109/ECC.2015.7330939). Posted with permission.

# Distributed Traffic Control for Reduced Fuel Consumption and Travel Time in Transportation Networks

Ran Dai, Jing Dong and Anuj Sharma

**Abstract**—This paper proposes a distributed framework for optimal control of vehicles in transportation networks. The objective is to reduce the balanced fuel consumption and travel time through hybrid control on speed limit and ramp metering rate. The dual decomposition theory associated with the subgradient method is then applied in order to decompose the optimal control problem into a series of suboptimal problems and then solve them individually via networked road infrastructures (RIs). Coordination among connected RIs is followed in each iteration to update the individual controls. An example is demonstrated to verify the reduction in terms of fuel consumption and travel time using the proposed approach.

**Index Terms**—Distributed Traffic Control, Eco-driving, Vehicle Fuel Consumption, Transportation Networks

## I. INTRODUCTION

Today's transportation system is experiencing a revolution that significantly impacts people's lives. The increasing demands of safety, traffic capacity and energy efficiency impelled the emergence of new technologies. Examples of such technologies include vehicle sensing, hybrid drive systems, communication networks, etc., all aimed at enhancing the public desired performance in terms of efficiency and safety. It can be envisioned that the effectiveness of vehicle guidance and traffic management strategies will be amplified through Dedicated Short Range Communication wireless networks. These networks will operate among vehicles and between vehicles and roadway infrastructure (RI), named vehicular networks [1], [2]. In this venue, not only the drivers in vehicles have direct access to dynamic traveling related information, but also RI has more accurate situational awareness.

Most existing traffic management infrastructures aim at alleviating congestion, where savings on vehicle fuel consumption is a byproduct of the regulation strategies. If fuel consumption is considered in evaluating the transportation system performance, it is necessary to analyze the effectiveness of current traffic control systems in terms of energy efficiency while guaranteeing the accomplishment of transportation tasks in desired time. Existing eco-driving strategies aiming at reducing fuel consumption in large transportation networks have not included the dynamic traffic flow model which characterizes the evolution of traffic flow velocity and density [3]–[5]. Although a centralized control algorithm has been developed to reduce balanced fuel consumption and travel time based on a second-order macroscopic traffic flow model, METANET, this new technique is limited to small scale transportation systems, i.e., one major road, due to the limited information and complexity of solving large scale optimization problems [6], [7].

In this paper, inspired by the recent breakthrough in distributed optimization techniques [8], [9], a distributed framework is applied

to optimize the performance of balanced fuel consumption and travel time through hybrid control on speed limits and traffic flow meters. The RIs with sensing, communication, and parallel computation functionalities, meet the requirements of carrying out the macroscopic level solution in the distributed approach. Going beyond the existing distributed architectures where precise dynamic flow models and fuel consumptions have not been included [10]–[12], work in this paper improves existing distributed strategies to generate real-time macroscopic level solutions with high precision.

The dual decomposition methodology and the associated subgradient method [13] have been applied in many optimization problems where each component in the dual function is a strict convex/concave problem and can be solved independently. The subgradient method, initiated by Shor [14] in 1970s, can then drive the dual function to a convergent solution by updating the dual variables which is related to the affine of the network in each iteration. Zelazo *et al.* [15] have applied the dual decomposition algorithm in distributed power management where schedule of each component can be controlled individually in heterogeneous power systems. Each suboptimal problem in their formulation is solved via dynamic programming and their solution is coordinated in a communication network to satisfy power constraints. In this paper, we implement the dual decomposition in the distributed traffic control network focusing on parallel computation and coordination among networked RIs.

In the following, we will first introduce the formulation of traffic control problems with traffic flow model, balanced fuel consumption and travel time model in §II. Then we proceed to present the distributed traffic control framework in transportation networks based on dual decomposition and subgradient method in §III. Simulation results using the proposed algorithm and relative performance evaluation are presented in §IV. Finally, concluding remarks are addressed in §V.

## II. PROBLEM FORMULATION

### A. METANET Traffic Flow Model

The METANET traffic flow model, introduced by Messmer and Papageorgiou in 1990 [16], describes the average dynamic behavior of vehicles in transportation networks. This macroscopic level traffic flow model is based on the continuous conservation law in the form of partial differential equations [17]. The additional constraints include speed dynamics, the initial traffic flow states, limits on flow density and speed, and the traffic flow conservation at junctions. The METANET model is applicable to freeway networks with arbitrary topology. Furthermore, the extended version of METANET allows integration of traffic control maneuvers, such as speed limits, ramp metering, and route planning [18], [19].

Assuming a transportation network is composed of  $M$  links, where each link is decomposed into  $N_m$  segments. For segment  $i$ ,  $i = 1, \dots, N_m$ , of link  $m$ ,  $m = 1, \dots, M$ , the flow, density, and space mean speed are presented by  $q_{m,i}$ ,  $\rho_{m,i}$ , and  $v_{m,i}$ ,

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respectively, as demonstrated in Figure 1. The dynamic evolution of state variables  $\mathbf{x}^T = [q_{m,i}, \rho_{m,i}, v_{m,i}]$  from time interval  $k$  to  $k+1$  are governed by

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{\Delta t}{l_m \gamma_m} [q_{m,i-1}(k) - q_{m,i}(k)] \quad (2.1)$$

$$v_{m,i}(k+1) = v_{m,i}(k) + \frac{\Delta t}{\tau} [V[\rho_{m,i}(k)] - v_{m,i}(k)] + \frac{\Delta t(v_{m,i-1}(k) - v_{m,i}(k))}{l_m} - \frac{\Delta t \eta (\rho_{m,i+1}(k) - \rho_{m,i}(k))}{\tau l_m (\rho_{m,i}(k) + \kappa)} \quad (2.2)$$

$$q_{m,i}(k) = \gamma_m \rho_{m,i}(k) v_{m,i}(k) \quad (2.3)$$

$$V[\rho_{m,i}(k)] = v_{free,m} \exp[-\frac{1}{a_m} (\frac{\rho_{m,i}(k)}{\rho_{cr,m}})^{a_m}], \quad (2.4)$$

where  $\gamma_m$  and  $l_m$  denote the number of lanes and length of link  $m$ , respectively.  $V[\rho_{m,i}]$  is the desired speed of segment  $i$  of link  $m$ ,  $v_{free,m}$  is the free flow speed,  $\tau$  is a time constant,  $\rho_{cr,m}$  is the critical density,  $a_m$  and  $\kappa$  are model parameters, and  $\Delta t$  is time step between  $k$  and  $k+1$ . The anticipation constant  $\eta$  is dependent on segment density and is set by two parameters,  $\eta_h$  and  $\eta_l$ , in the form of

$$\eta = \begin{cases} \eta_h, & \text{if } \rho_{m,i+1}(k) \leq \rho_{m,i}(k) \\ \eta_l, & \text{otherwise.} \end{cases} \quad (2.5)$$

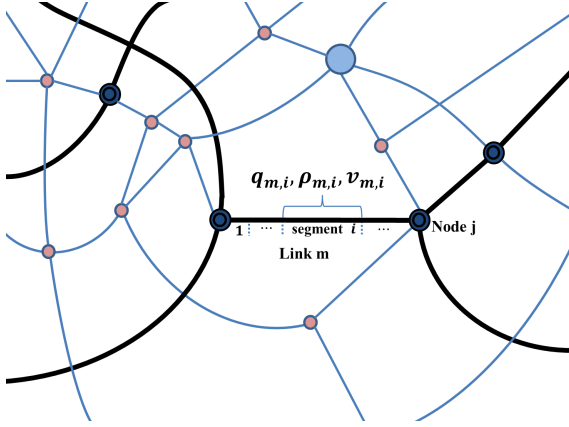


Fig. 1. Example of traffic networks

The demand at the origin of link  $m$ ,  $m = 1, \dots, M$ , is specified by  $d_{m,o}(k)$ , which is estimated according to observed traffic history. The outflow at the origin,  $q_{m,o}$ , is determined by

$$q_{m,o}(k) = \min\{d_{m,o}(k) + \frac{w_{m,o}(k)}{\Delta t}, C_{m,o}, C_{m,o}(\frac{\rho_{jam,m} - \rho_{m,1}(k)}{\rho_{jam,m} - \rho_{cr,m}})\}, \quad (2.6)$$

where  $C_{m,o}$  is the flow capacity at the origin,  $\rho_{jam,m}$  is the maximum density of link  $m$ , and  $w_{m,o}(k)$  is the flow of waiting queue at origin of link  $m$ , which is modeled by

$$w_{m,o}(k+1) = w_{m,o}(k) + \Delta t(d_{m,o}(k) - q_{m,o}(k)). \quad (2.7)$$

Moreover, the downstream density  $\rho_{m,N_m+1}$  is considered to be congestion free. Thus it is the smallest of the critical density,  $\rho_{cr,m}$ , and the density of the last segment,  $\rho_{m,N_m}(k)$ , expressed as

$$\rho_{m,N_m+1}(k) = \min\{\rho_{m,N_m}(k), \rho_{cr,m}\}. \quad (2.8)$$

When the destination demand is specified by  $\rho_d(k)$ , the above virtual downstream density is simply set as  $\rho_{m,N_m+1}(k) = \rho_d(k)$ . Furthermore, at a connection node, the total flow enters node  $j$  is equivalent to the total flow leaving the same node, expressed as

$$\sum_{s_j \in S_j} q_{s_j,N_{s_j}}(k) = \sum_{u_j \in U_j} q_{u_j,o}(k), \quad j = 1, \dots, J, \quad (2.9)$$

where the link set  $s_j \in S_j$  represents the links entering node  $j$ , the link set  $u_j \in U_j$  represents the links leaving node  $j$ , and  $J$  is the total number of nodes/junctions. The detailed description of METANET model and assumptions on origins and downstream can be referred to [7].

When control variables are considered in the traffic dynamic model, the METANET introduced in Eqs. (2.1)-(2.9) for evolution of free flow has an extended version. Assume segment set  $i \in i_c$  of link  $m$  has control on speed limit at time interval  $k$ , denoted as  $V_{limm,i_c}(k)$ , the expression of desired speed in Eq. (2.4) is updated by

$$V[\rho_{m,i_c}(k)] = \min\{v_{free,m} \exp[-\frac{1}{a_m} (\frac{\rho_{m,i_c}(k)}{\rho_{cr,m}})^{a_m}], V_{limm,i_c}(k)\}. \quad (2.10)$$

Another type of control variable is the ramp metering rate at origin of a link. Assuming link  $m_c$  has ramp meter at the origin, the metering rate at time interval  $k$  is denoted as  $r_{m_c,o}(k)$ . Under this assumption, the outflow entering into link  $m_c$  in Eq. (2.6) is updated by

$$q_{m_c,o}(k) = \min\{d_{m_c,o}(k) + \frac{w_{m_c,o}(k)}{\Delta t}, r_{m_c,o}(k) C_{m_c,o}, C_{m_c,o}(\frac{\rho_{jam,m_c} - \rho_{m_c,1}(k)}{\rho_{jam,m_c} - \rho_{cr,m_c}})\}. \quad (2.11)$$

The other equations are in the same form of free flow model.

### B. Balanced Fuel Consumption and Travel Time Model

At microscopic level, instantaneous vehicle fuel consumption can be estimated using regression models, such as VT-Micro [20], or power-demand models, such as Comprehensive Modal Emission Model [21]. In this paper we adopt VT-Micro model, which was validated against real world field data and other emissions models [22]. The instantaneous fuel consumption for segment  $i$  of link  $m$  over time interval  $k$  using the VT-Micro model is estimated based on relative speed  $v_{m,i}(k)$  and acceleration  $a_{m,i}(k)$  in the form of

$$J_{f,m}(k) = \sum_{i=1}^{N_m} \exp \tilde{v}_{m,i}^T(k) P_f \tilde{a}_{m,i}(k), \quad (2.12)$$

where  $P_f$  is a pre-defined  $4 \times 4$  parameter matrix,  $\tilde{v}_{m,i} = [1, v_{m,i}, v_{m,i}^2, v_{m,i}^3]^T$ ,  $a_{m,i}(k) = (v_{m,i}(k+1) - v_{m,i}(k))/\Delta t$ , and  $\tilde{a}_{m,i} = [1, a_{m,i}, a_{m,i}^2, a_{m,i}^3]^T$ .

The total travel time of all the vehicles in link  $m$  and waiting queue over time interval  $k$  includes the time spent traversing the roadway segment and the waiting time in queue, expressed as

$$J_{t,m}(k) = \sum_{i=1}^{N_m} (l_m \gamma_m \rho_{m,i}(k) + w_o(k)) \Delta t. \quad (2.13)$$

When considering balance fuel consumption and travel time, the two objectives in Eqs. (2.12) and (2.13) are normalized by the respective nominal values, i.e.  $J_{f,norm}$  and  $J_{t,norm}$ , corresponding to the fuel consumption and total travel time in the uncontrolled scenario, and then weighted using a weighting factor  $\theta$ , which is an empirical parameter to benefit a majority of drivers within the

network. The balanced objective function of the entire transportation network over time  $k = 1$  to  $k = K$  is determined by

$$J = \sum_{k=1}^K \sum_{m=1}^M \left( \frac{J_{f,m}(k)}{J_{f,norm}} + \theta \frac{J_{t,m}(k)}{J_{t,norm}} \right). \quad (2.14)$$

### C. Traffic Control Problem Formulation

As a summary, the traffic control problem is to find the hybrid control variables  $V_{lim,m,i_c}(k)$  and  $r_{m_c,o}(k)$  to minimize the balanced fuel consumption and travel time while satisfying the dynamics of traffic states. The mathematical formulation of the optimal control problem is expressed as

$$\begin{aligned} J &= \min_{V_{lim,m,i_c}(k), r_{m_c,o}(k)} \sum_{k=1}^K \sum_{m=1}^M \left( \frac{J_{f,m}(k)}{J_{f,norm}} + \theta \frac{J_{t,m}(k)}{J_{t,norm}} \right) \\ \text{s.t. } \mathbf{x}(k+1) &= f(\mathbf{x}(k)) \\ \mathbf{x}_{m,0}(k) &= f_s(\mathbf{x}(k)), m = 1, \dots, M \\ \mathbf{x}_{m,N_m+1}(k) &= f_e(\mathbf{x}(k)), m = 1, \dots, M \\ \sum_{s_j \in S_j} q_{s_j, N_{s_j}}(k) &= \sum_{u_j \in U_j} q_{u_j, o}(k), j = 1, \dots, J. \end{aligned} \quad (2.15)$$

where  $f$  represent the dynamics function described in Eqs. (2.1)-(2.4),  $f_s$  represents the boundary condition of origin expressed in Eqs. (2.6) and (2.7), and  $f_e$  represents the boundary constraint at downstream in Eq. (2.8). With minor adaption, the above problem can be transformed into an optimization problem to minimize fuel consumption under the constraint of total travel time, denoted as  $\sum_{k=1}^K \sum_{m=1}^M J_{t,m}(k) \leq J_{t,max}$ , where  $J_{t,max}$  is the threshold of the total travel time.

## III. DISTRIBUTED TRAFFIC CONTROL FRAMEWORK

In this section, we will proceed with the distributed optimization method. We will firstly look into the framework of the distributed traffic control and then introduce the dual decomposition and subgradient method which is applicable to solve this type of decomposable subproblems with constraints. The detailed algorithm is summarized in the following subsection.

### A. Framework

A transportation network shown in Figure 1 can be represented as a directed graph  $\mathcal{G}_T = (V_T, \mathcal{E}_T)$ , composed of the vertex set  $V_T$  with cardinality  $J$ , representing the  $J$  nodes/junctions, and edge set  $\mathcal{E}_T$ , representing the  $M$  links that connect two element subsets of  $V_T$ . In such connected transportation network, we assume each RI of link  $m$  can communicate information with its neighboring RIs whose link is joined at the same node, as demonstrated in Figure 2. There is no central processor and the network is not fully connected. However, as long as one RI is connected to at least one other RI in the network, the two connected RIs can communicate information with each other. Furthermore, they are able to spread the information among the connected network finally. In such a system, the centralized optimization or control algorithm that requires fully connected network cannot be applied in such partially connected system. In addition, the fully connected network requires large data communication and storage which may bring difficulty for implementation with the increase of data numbers.

### B. Dual Decomposition and Subgradient Method

For a nonlinear optimization problem of minimizing the objective function  $f(x)$  under equality constraints  $h_i(x) = 0 (i = 1, \dots, p)$

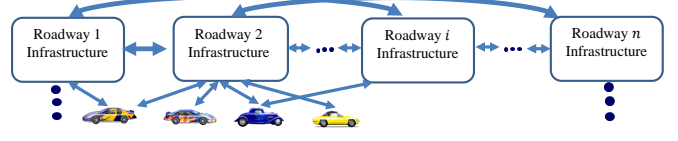


Fig. 2. Distributed traffic control framework

and inequality constraints  $g_i(x) \leq 0 (i = 1, \dots, m)$ , its Lagrangian has the form:

$$L(x, \lambda, \mu) = f(x) + \sum_{i=1}^p \mu_i h_i(x) + \sum_{i=1}^m \lambda_i g_i(x). \quad (3.16)$$

If the objective and constraint functions can be expressed in the summation form as

$$f(x) = \sum_{k=1}^K f^k(x^k), g_i(x) = \sum_{k=1}^K g_i^k(x^k), h_i(x) = \sum_{k=1}^K h_i^k(x^k),$$

by partition the state vector  $x$  into subvectors  $x = (x^1, \dots, x^K)$ , then the Lagrangian is reformulated as

$$L(x, \lambda, \mu) = \sum_{k=1}^K (f^k(x^k) + \sum_{i=1}^p \mu_i h_i^k(x^k) + \sum_{i=1}^m \lambda_i g_i^k(x^k)).$$

The above function can be decomposed into  $K$  subproblems according to the subvector  $x^k$ , where  $k = 1, \dots, K$ . For each subproblem, it can be solved by minimizing the dual function

$$L_D^k(\lambda, \mu) = \min_{x^k} (f^k(x^k) + \sum_{i=1}^p \mu_i h_i^k(x^k) + \sum_{i=1}^m \lambda_i g_i^k(x^k)) \quad (3.17)$$

for a given pair of multipliers  $(\lambda^k, \mu^k)$ . The dual function defined as

$$L_D^k(\lambda, \mu) = \inf_x L(x, \lambda, \mu) \quad (3.18)$$

is always concave, thus the dual problem of the summation of the subproblems

$$\max_{(\lambda, \mu)} L_D^k(\lambda, \mu) \quad (3.19)$$

is a convex optimization problem and can be solved by the subgradient method.

The subgradient method is an iterative procedure to gradually process the optimization solution by finding the ascent direction for the dual problem. At each sequence  $j$ , assuming the multipliers  $(\lambda_j, \mu_j)$  are given, the subgradient at this point is expressed as

$$d_j = \begin{pmatrix} g(x_j) \\ h(x_j) \end{pmatrix}. \quad (3.20)$$

Then the multipliers are updated as follows:

$$\begin{aligned} \lambda_{j+1} &= \max(0, \lambda_j + \alpha_j g(x_j)) \\ \mu_{j+1} &= \mu_j + \alpha_j h(x_j), \end{aligned} \quad (3.21)$$

where  $\alpha_j$  is the step size that will control the convergent speed of the subgradient method. The maximum number of sequence  $j$  is generally defined to satisfy the stopping criteria of iteration.

The new algorithm proposed in this paper is expected to build a distributed framework to search for a precise optimal solution by communicating between connected RIs without central processor. In the following, we will explain how to solve the subproblems via individual RIs and to coordinate them through subgradient method.

### C. Solution of Subproblem

Without the flow conservation at junctions in Eq. (2.9), the traffic control problem of the entire transportation network, formulated in Eq. (2.15), can be decomposed into a series of subproblems for each controlled link. For example, if link  $m$  is under control of limited speed of segment set  $\{i_c\}$ , the subproblem is formulated as

$$\begin{aligned} J &= \min_{V_{i_m m, i_c}} \sum_{k=1}^K \left( \frac{J_{f_m}(k)}{J_{f, norm_m}} + \theta \frac{J_{t_m}(k)}{J_{t, norm_m}} \right) \\ \text{s.t. } \mathbf{x}_m(k+1) &= f(\mathbf{x}_m(k)) \\ \mathbf{x}_{m,0}(k) &= f_s(\mathbf{x}(k)), \mathbf{x}_{m, N_m+1}(k) = f_e(\mathbf{x}(k)). \end{aligned} \quad (3.22)$$

However, due to the high nonlinearity and involvement of logical constraints in the above optimal control problem, each subproblem is classified as nonconvex and nonlinear. Therefore, solving each subproblem with guaranteed global optimality is challenging. In real world, the speed limit signals in highways are set in different scales with 10 km/h or 5 km/h difference between two adjacent scales. Based on this fact, we set the control variables of the speed limit as a series of discrete numbers, i.e., [10, 20, ..., 110, 120] km/h. Assuming the control of speed limit has  $n_c$  scales at control segment  $i$ , the propagation of states  $\mathbf{x}_m$  from  $k$  to  $k+1$  has accordingly  $n_c$  transition options. For the same reason, the propagation of states after time interval  $k+1$  has  $n_c$  transition options as well. In general,  $n_c$  control scales will generate the same number of state sequences in time frame. The objective function for each sequence can be calculated based on the propagated states.

When road  $m$  has  $I_c$  number of segments under controlled speed limit, there will be  $n_c^{I_c}$  state sequences and associated cost for each sequence. We are searching for the optimal combination of the  $I_c$  control variables that will lead to the minimum total cost from time  $k=1$  to  $k=K$ . For road with metering rate control, we can make the same assumption that the metering rate value is discretized into several scales within allowed range. The objective is to search for the optimal discrete controls as well. The key observation here is the discrete nature of the states and decision variables. With such a perspective, tools from dynamic programming can be used. More intuitive is the use of shortest path algorithms to solve the subproblems. Given the state-transition relationship and assigned cost to each transition, an appropriate algorithm such as Dijkstras or Bellman-Ford algorithm, can be used to solve this subproblem [23].

### D. Coordination among RIs

As we introduced in the dual decomposition and subgradient method, during time interval  $k$ , the Lagrangian function for the traffic control problem in (2.15) is represented by

$$\begin{aligned} L &= \sum_{k=1}^K \left[ \sum_{m=1}^M \left( \frac{J_{f_m}(k)}{J_{f, norm}} + \theta \frac{J_{t_m}(k)}{J_{t, norm}} \right) + \right. \\ &\quad \left. \sum_{j=1}^J \lambda_j \left( \sum_{s_j \in S_j} q_{s_j, N_{s_j}}(k) - \sum_{u_j \in U_j} q_{u_j, N_{u_j}}(k) \right) \right], \end{aligned} \quad (3.23)$$

where the multiplier  $\lambda_j$ ,  $j=1, \dots, J$ , is associated with the flow conservation constraints at  $j$  node. By sorting  $q_{s_j, N_{s_j}}$  and  $q_{u_j, N_{u_j}}$  in Eq. (3.23), the Lagrangian function can be rewritten as

$$\begin{aligned} L &= \sum_{m=1}^M \sum_{k=1}^K \left[ \frac{J_{f_m}(k)}{J_{f, norm}} + \theta \frac{J_{t_m}(k)}{J_{t, norm}} + \lambda_{j_{m_u}} q_{m, N_m}(k) \right. \\ &\quad \left. - \lambda_{j_{m_s}} q_{m, o}(k) \right], \end{aligned} \quad (3.24)$$

where the origin of link  $m$  is connected at node  $j_{m_s}$  and the downstream of link  $m$  is connected at node  $j_{m_u}$ . From the above

reformatted expression, it is easy to tell that Eq. (3.24) is composed of  $M$  subproblems which can be summarized as

$$\begin{aligned} L_m &= \min_{V_{i_m m, i_c}(k), r_{m, o}(k)} \sum_{k=1}^K \left[ \frac{J_{f_m}(k)}{J_{f, norm}} + \theta \frac{J_{t_m}(k)}{J_{t, norm}} \right. \\ &\quad \left. + \lambda_{j_{m_u}} q_{m, N_m}(k) - \lambda_{j_{m_s}} q_{m, o}(k) \right], m=1, \dots, M. \end{aligned} \quad (3.25)$$

For given Lagrangian multipliers  $\lambda_{j_{m_s}}$  and  $\lambda_{j_{m_u}}$ , the above subproblems are independent of each other with its own states and control variables. Therefore they can be solved individually using the method discussed in §III-C. After a careful comparison of these subproblems with the local subproblem in Eq. (3.22), we found that they are very similar to each other except the additional term  $(\lambda_{j_{m_u}} q_{m, N_m}(k) - \lambda_{j_{m_s}} q_{m, o}(k))$ . However, the controls need to be recalculated to obtain the new optimal solution for each subproblem with the additional term in the original objective function. By resetting the cost of state transition using the newly formulated objective in Eq. (3.25), each subproblem can be solved through dynamic programming.

The calculation of each subproblem can be performed by each RI providing the lagrangian multipliers and its local traffic information. Comparing the updated objective function in distributed framework with the original formulation in Eq. (3.22), the additional term in Eq. (3.25) comes from the lagrangian multipliers. This extra term works as an coordinating parameter for local computation of each RI to adjust its solution to meet the constraint of flow conservation at corresponding junction. Using the subgradient method, the lagrangian multipliers are updated by expressions in Eq. (3.21).

The states  $q_{m, N_m}$  and  $q_{m, o}$  and lagrangian multipliers from the connected links are coupled with each other. The iterations are repeated procedure of adjustment and comparison. The flow conservation constraints are re-evaluated by adjusted states from neighborhood links in the subgradient method to find the new adjustment in the next loop. Ideally, the conservation constraint at each node will converge to equality. The stopping criteria for the convergent solution can use a fixed maximum iteration number or set the sum of least square difference between downstream and leaving stream less than a threshold. At every iteration step, each RI will calculate its own control variables based on the new optimal formulation. Then information exchange will be performed among neighboring RIs to obtain the updated multipliers in the objective term. Different from the centralized optimization framework that requires all RIs to communicate with the central processor with updated information, i.e., density, flow, average velocity, et al., as the public variables, the distributed framework only requires information of the downstream flow, outflow at the origin, and lagrangian multipliers. Furthermore, the distributed framework can carry out parallel calculation by individual RI, which will save the overall calculation time.

## IV. SIMULATION EXAMPLE

In this section, we will apply the distributed framework in a real scenario with hybrid traffic control on two major roads, similar to the example presented in [7]. As demonstrated in Figure (3), link one is a two-lane 12 km highway and link two is a one-lane 6 km highway. The two roads are joined at their ending points. The relative parameters used in the simulation are provided in Table I. In addition, segment length of both roads are set as 1 km. Speed limit control signals for road one are set at the starting point of segment 3 and 7. Road two has traffic flow meter control at the origin. The demand at two origins and downstream density with respect to time are specified in Figures 4(a) and 4(b), respectively. Moreover, we assume the leaving link at conjunction is a three-lane road with

capacity of 2000  $veh/h/lane$  such that  $q_{1,12}(k) + q_{2,6}(k) \leq 6000$ ,  $\forall k = 1, \dots, K$ .

TABLE I  
PARAMETERS USED IN SIMULATION EXAMPLE

$\kappa(veh/km/lane)$	$\eta_h(km^2/h)$	$\tau(sec)$
40	50	18
$\rho_{cr,m}(veh/km/lane)$	$\eta_l(km^2/h)$	$\Delta t(sec)$
33.5	42	10
$\rho_{jam,m}(veh/km/lane)$	$v_{free,m}(km/h)$	$a_m$
180	102	1.867
$C_{o,1}(veh/h/lane)$	$C_{o,2}(veh/h/lane)$	$\theta$
2000	2000	5

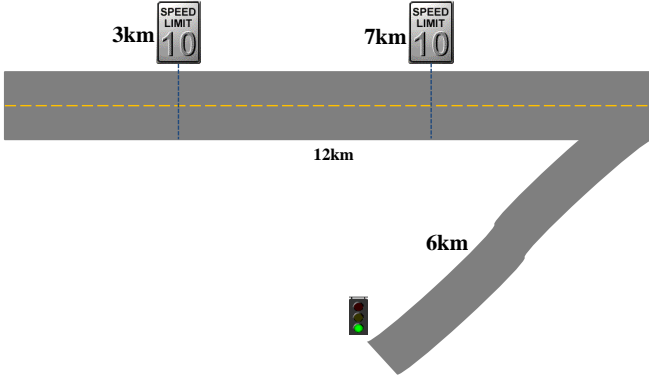
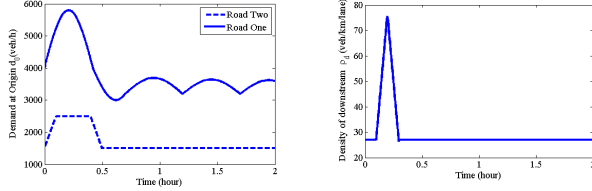


Fig. 3. Scenario of simulation example



(a) Demand at origins of road one and two (b) Downstream density at junction

Fig. 4. Demand at origins and downstream density.

The simulation period for this example lasts for 2 hours. The designed control variables will be updated every 20 minutes and are assumed to be constant under each controlled interval. We implement the distributed framework proposed above in this example and the simulation results of the objective value with and without controls are provided in Table II. Furthermore, we provide the density history of each segment for both roads in Figures 5-8. Figure 5 shows that the high demands at the origin and downstream of link one will cause congestion, where the density will be above 100  $veh/km/lane$  close to segment 2 after 1 hour, under the scenario of free flow/no controls. Similar congestion is generated in link two without meter control, as demonstrated in Figure 7. The control commands, as shown in Table III, effectively mitigate congestion in the transportation network due to origin and outflow peaks. The computed speed limit control signals have low speed, 45  $km/h$ , at segments 3 and 7 in the first time interval and slightly increased speed limit, 60  $km/h$ , in the following one hour. Traffic following

these control signals allows more vehicles pass through link one without causing congestion due to the peak demands in initial time interval at the origins and outflow. The same principal applies to link two where the controlled rate at the first two time intervals is 50%. From the simulation results, we obtain significant reduction in both fuel consumption and travel time, as indicated in Table II.

TABLE II  
FUEL CONSUMPTION AND TRAVEL TIME IN SIMULATION EXAMPLE WITH AND WITHOUT CONTROLS

Road	$J_f$ with control	$J_f$ without control	reduction percentage
1	8245.81 kg	8925.56 kg	7.62%
2	1877.42 kg	2090.51 kg	10.19%
Road	$J_t$ with control	$J_t$ without control	reduction percentage
1	1778.01 $veh \cdot h$	2100.83 $veh \cdot h$	15.36%
2	373.07 $veh \cdot h$	469.6 $veh \cdot h$	20.57%

TABLE III  
CONTROL HISTORY OF SPEED LIMIT AND RAMP RATE

Time Interval (mins)	20	40	60	80	100	120
speed limit (km/h)	45	60	60	60	60	85
ramp rate (%)	50	50	100	100	100	100

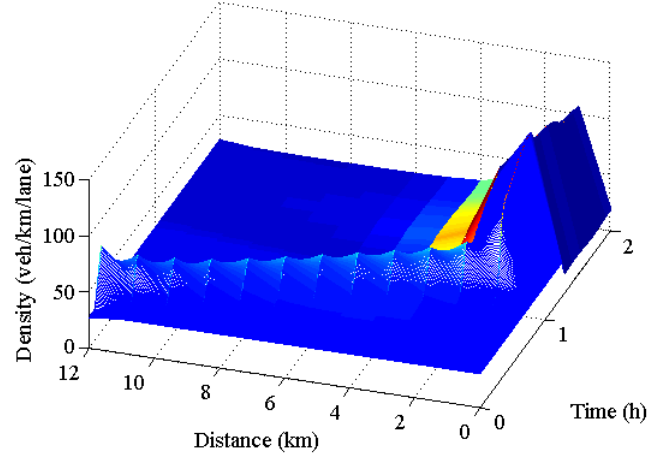


Fig. 5. Density history of road one without control

## V. CONCLUSIONS

In this paper a novel way to solve traffic control problems for balanced fuel consumption and travel time was proposed based on dual decomposition and subgradient method. A distributed framework was constructed to design hybrid traffic control variables of speed limit and metering rate in a transportation network which was modeled via precise nonlinear dynamics. The dynamic programming algorithm was applied to solve the decomposed subproblems. The distributed dual-decomposition algorithm then approached the optimal solution with linear constraints in an iterative manner. Simulation results verified significant reduction in both fuel consumption and travel time.



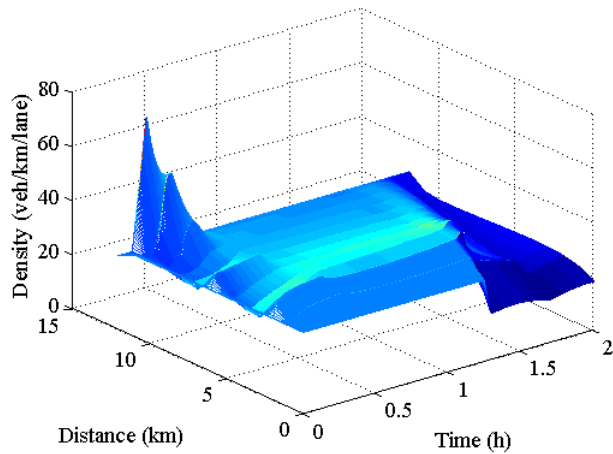


Fig. 6. Density history of road one with control

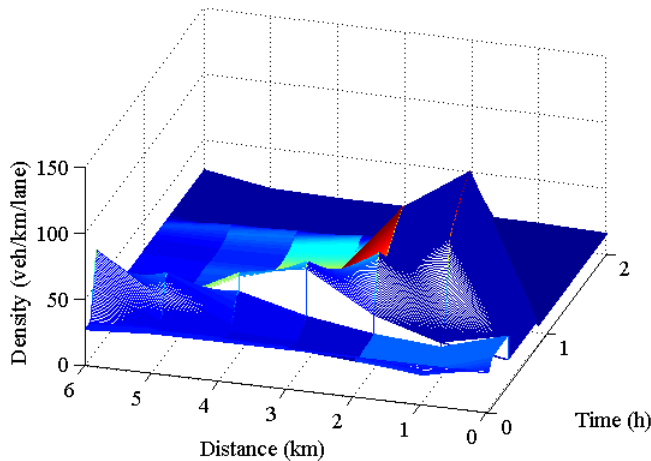


Fig. 7. Density history of road two without control

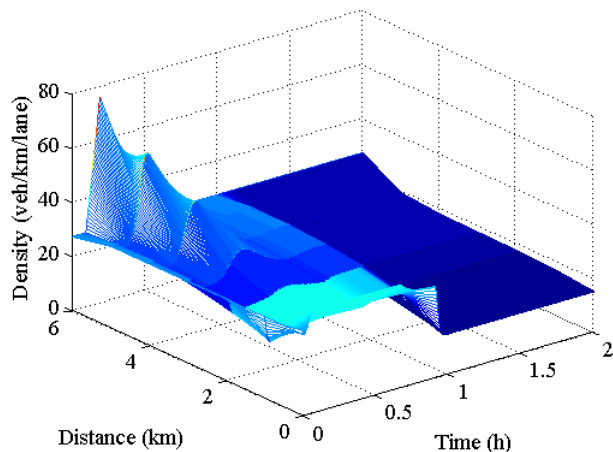


Fig. 8. Density history of road two with control

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